

CCGPS Frameworks Student Edition

Mathematics

7th Grade Unit 4: Inferences



Dr. John D. Barge, State School Superintendent "Making Education Work for All Georgians"

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<u>Unit 4:</u> Inferences

Task

Is it Valid? Candy Population Predicting Population Counting Trees (FAL) Shakespeare vs. Harry Potter Got Friends? Travel Times to Work

STANDARDS ADDRESSED IN THIS UNIT

MCC7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

MCC7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

MCC7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

MCC7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

STANDARDS FOR MATHEMATICAL PRACTICE

Make sense of problems and persevere in solving them. Students make sense of information by connecting visual, tabular, and symbolic representations of sample populations in real-life contexts.
*2. Reason abstractly and quantitatively. Students' reason about the values in data representations based upon their relationship to the real number line.

***3.** Construct viable arguments and critique the reasoning of others. Students use data to make inferences from sample sets. They construct viable arguments by referring to representations as evidence of their inferences and question each other regarding these inferences.

***4. Model with mathematics.** Students generate representative samples in real-world contexts and represent these visually, in tables, and symbolically to gain information from sample sets.

5. Use appropriate tools strategically. Students choose appropriate mathematical and visual representations, including technology-based tools, to represent the data distributions.

6. Attend to precision. Students use precision to collect accurate measurement information from sample populations and precise language when generating and interpreting data.

7. Look for and make use of structure. Students interpret data representations in tables, histograms, box plots and scatter plots by examining the features of those representations.

8. Look for and express regularity in repeated reasoning. Students look to make generalized comparisons between situations that involve bias using specific criteria.

Is it Valid?

As you enter the room, please complete the five survey questions posed by your teacher.

PART I

A survey is a method of collecting information. These pieces of information, numbers and facts, are called data.

DESCRIBE YOUR CLASS SURVEY In the space below, compile the data collected by your classmates.

| 1. | What color is your hair? Brown Blond Red Black |
|----|---|
| 2. | Do you carry a cell phone? Yes No |
| 3. | About how many text messages do you send in a day? 0-20 21-40 >40 |
| 4. | About how much time do you spend on homework? |
| | 0-15 min 16-30 min 31-45 min 46-60 min > 60 min |
| 5. | What month were you born? |
| | Jan – March April – June July-Sept Oct-Dec |

Using the survey results discuss the following vocabulary.

- 6. What is the population of your survey?
- 7. Did your class perform a census or a survey?
- 8. How would you record your results from question 1 as a piece of data?
- 9. How would you describe your results from question 2 as a parameter for the population?

For each of the problems below, answer the questions in regard to population, samples, and surveys.

After the 2000 census, the United States Census Bureau reported that 7.4% of Georgia residents were between the ages of 10 and 14.

- 10. Was a parameter or a statistic reported? Explain your reasoning.
- 11. What is the population for this survey?

The manager of the Millcreek Mall wants to know the mean age of the people who shop at the mall and the stores in which they typically shop. He hires Barker Market Research Company to collect the data. Dennis works for the Barker Market Research Company and has been put in charge of collecting data for the Millcreek Mall. Dennis decides to interview 100 people one Saturday because it is the mall's busiest shopping day.

- 12. Is the survey a sample or a census?
- 13. What is the population for this survey?
- 14. Is the sample size for the survey appropriate?

PART II There are three key things to look for when determining if a survey is biased.

- 15. Is the survey voluntary?
- 16. Does the survey exclude any group?
- 17. Is the sample size appropriate?

In the summaries below, determine if the sample taken is representative of the population, without bias shown:

- 18. ABC Family is a television channel that targets families and young adults to view their station. ABC Family regularly posts online poll questions to their website. In 2010, ABC Family polled their viewers to ask about airing Rated "R" movies after 8pm on their channel. Almost 200,000 people responded, and 85% of them disagreed with airing Rated "R" movies.
- 19. Mrs. Jones wants to know how the 5th grade feels about recess time. Mrs. Jones labels every student in the 5th grade with a number. She then draws 50 numbers out of a hat and surveys these students. Mrs. Jones determines that 5th graders would like more recess time than they currently have.

20. The City of Smallville wants to know how its citizens feel about a new industrial park in town. Surveyors stand in the Smallville Mall from 8am-11am on a Tuesday morning and ask people their opinion. 80% of the surveyed people said they disagreed with a new industrial park.

21. The National Rifle Association (NRA) took a poll on their website, <u>www.nra.com</u>, and asked the question, "Do you agree with the 2nd Amendment: the Right to Bear Arms"? 98% of the people surveyed said "Yes", and 2% said "No".

Candy Populations

1. Count the number of EACH color of candies and record your answers in a frequency table. Make sure to label your frequency table clearly.

 Using the results from your group, fill in the following chart and find the number of M&Ms for each person in your group and the total number of M&Ms. Find the percentage of each color of M&Ms for your sample.

| Candy Color | Number of candies in Sample # 1 | Number of candies in Sample # 2 | Number of candies in Sample # 3 | Number of candies in Sample # 4 | Number of candies in Sample # 5 | Number of candies in Sample # 6 | Total Number of candies in ALL Samples | % of Each Color of candies |
|---------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---|----------------------------------|
| Red | | | | | | | | |
| Orange | | | | | | | | |
| Yellow | | | | | | | | |
| Green | | | | | | | | |
| Blue | | | | | | | | |
| Brown | | | | | | | | |
| Total # of M&Ms in Sample | | | | | | | | |

Answer the following questions about your sample:

- 3. Is your sample random? Explain your answer.
- 4. Do you think your sample could predict the number of M&Ms in a larger bag? Why or why not?
- 5. Do you think that the percentages of each color of M&Ms are the same as other groups? Why or why not?
- 6. Using your data values, estimate the number of M&Ms for EACH color for a bag of 1000 candies. Record your data in the table below.

| Color | Estimate for Number of M&Ms in a Population of 1000 M&Ms |
|--------|--|
| Red | |
| Orange | |
| Yellow | |
| Green | |
| Blue | |
| Brown | |

Predicting Populations: Goldfish Lab

You have a bag with fish crackers in it. We are going to "tag" a sample of the fish and make a prediction about the total population of fish found in the bag.

- 1. Remove 10 crackers from the bag.
- 2. Tag them by marking on them with a marker.
- 3. Put the fish back in the bag and shake them up.
- 4. Remove 20 fish crackers.
- 5. Set up a proportion and make a prediction for how many fish are in the bag.

We will fill in the following chart based on the data from each group.

CLASS DATA TABLE

| Group Number | Number of tagged fish in the sample | Total number in the sample | Total number of tagged individuals in the population | Total Estimated Population |
|--------------|-------------------------------------|----------------------------|--|-------------------------------|
| One | | | | |
| Two | | | | |
| Three | | | | |
| Four | | | | |
| Five | | | | |
| Six | | | | |
| Seven | | | | |
| Eight | | | | |

- 1. What is the average estimated population for your class?
- 2. Count the number of fish in the bag. What was the actual population size?
- 3. Is this a census or a survey? Justify your response.

4. What is your percent of error based on the estimated population and actual population?

5. Does this method seem reliable for wildlife population? Explain.

6. If there is a population of 800 deer, what would be a good sample size? Justify your answer.

Formative Assessment Lesson: Counting Trees

This diagram shows some trees in a tree farm.

The circles 🔎 show old trees and the triangles 🔺 show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

- What method could he use to estimate the number of trees of each type? Explain your method fully.
- 2. On your worksheet, use your method to estimate the number of:
 - (a) Old trees
 - (b) Young trees

Shakespeare vs. Harry Potter

Many skeptics feel that there has been a "dumbing down" of America's youth of the past decades. To determine if there is any truth to this claim, we will compare two pieces of literature: Shakespeare's *Macbeth* and JK Rowling's *Harry Potter and the Chamber of Secrets*.

Is there a difference in the length of the words used in a Shakespeare play compared to a Harry Potter book? Today you will sample words from both pieces of literature to determine who used longer words.

Below are excerpts from a Shakespeare Novel and a Harry Potter book. Follow the steps below to determine which piece of literature uses longer words.

- 1. Roll a number cube once for each line of *Macbeth* below. Move to the word that corresponds to the number you roll (1 = first word in the line, 2 = second word in the line, etc.).
- 2. Count the letters in that word, and record the number in the table below in the "Letter Count" columns.
- 1. Is this a dagger which I see before me,
- 2. The handle toward my hand? Come, let me clutch thee!
- 3. I have thee not, and yet I see thee still.
- 4. Art thou not, fatal vision, sensible
- 5. To feeling as to sight, or art thou but
- 6. A dagger of the mind, a false creation,
- 7. Proceeding from the heat oppressed brain?
- 8. I see thee yet, in form as palpable
- 9. As this which now I draw.
- 10. Though marshal'st me the way I was going;
- 11. And such an instrument I was to use.
- 12. Mine eyes are made the fools o' th' other senses,
- 13. Or else worth all the rest. I see thee still;
- 14. And on thy blade and dudgeon gouts of blood,
- 15. Which was not so before. There's no such thing.

Shakespeare:

| Line Number | Letter Count | Line Number | Letter Count |
|-------------|-----------------|-------------|-----------------|
| 1 | | 9 | |
| 2 | | 10 | |
| 3 | | 11 | |
| 4 | | 12 | |
| 5 | | 13 | |
| 6 | | 14 | |
| 7 | | 15 | |
| 8 | | | |

- 3. Now, roll the number cube to see which words you will count in the Harry Potter excerpt. You will count the letters in two words from each line. Roll the die and move to the word that corresponds to that number. After counting the number of letters in that word, roll the dice again and move to another word in the same line. Then, move to the next line and repeat the process for each of the lines in the passage. Record your numbers in the chart that follows in the "Letter Count" columns.
- 1. October arrived, spreading a damp chill over the grounds and into the
- 2. castle. Madam Pomfrey, the nurse, was kept busy by a sudden
- 3. spate of colds among the staff and students. Her Pepperup potion
- 4. worked instantly, though it left the drinker smoking at the ears for
- 5. several hours afterward. Ginny Weasley, who had been looking
- 6. pale, was bullied into take some by Percey. The steam pouring from
- 7. under her vivid hair gave the impression that her whole head was on fire.

Harry Potter:

| Line Number | Letter Count Word 1 | Letter Count Word 2 |
|-------------|---------------------------|---------------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |

4. Find the mean (\bar{x}) of the Shakespeare word sample and the Harry Potter word sample. Do this by adding the letter counts for both passages, separately. Then divide the sum for each passage by the total number of words (15 for Shakespeare; 14 for Harry Potter).

Shakespeare mean $(\bar{x}) =$

Harry Potter mean (\bar{x}) =_____

- 5. Find the Mean Absolute Deviation of the Shakespeare data using the table below.
 - a) Find the distance that each value is away from the mean (\bar{x}) .

b) Determine the absolute value of each deviation from the mean.

c) Total the values from the Mean Absolute Deviation column.

Shakespeare:

| Line # | Letter Count | Deviation from Mean(\overline{x}) Letter Count- (\overline{x}) | $\frac{\text{Mean}(\bar{x}) \text{ Absolute Deviation}}{ \text{ Letter Count-}(\bar{x}) }$ |
|-----------|--------------|---|--|
| 1. | | | |
| 2. | | | |
| 3. | | | |
| 4. | | | |
| 5. | | | |
| 6. | | | |
| 7. | | | |
| 8. | | | |
| 9. | | | |
| 10. | | | |
| 11. | | | |
| 12. | | | |
| 13. | | | |
| 14. | | | |
| 15. | | | |
| | | | Total: |

d) Divide by the number of values counted

- 6. Find the Mean Absolute Deviation of the Harry Potter data using the table below.
 - a. Find the difference between each letter count value and the mean (\bar{x}) .
 - b. Determine the absolute value of each deviation from the mean.
 - c. Total the values from the Mean Absolute Deviation column.

Harry Potter:

| Line | Letter | Deviation from Mean (\overline{x}) | Mean(\overline{x}) Absolute Deviation |
|------|--------|---|---|
| # | Count | Letter Count- (\bar{x}) | Letter Count- (\bar{x}) |
| 1. | | | |
| 2. | | | |
| 3. | | | |
| 4. | | | |
| 5. | | | |
| 6. | | | |
| 7. | | | |
| 8. | | | |
| 9. | | | |
| 10. | | | |
| 11. | | | |
| 12. | | | |
| 13. | | | |
| 14. | | | |
| | | | Total: |

d. Divide the total by the number of values counted.

7. Find the five number summary using the data you found for Shakespeare and Harry Potter. Enter your data into the table below:

| Shakespeare | Harry Potter |
|-------------|--------------|
| Minimum | Minimum |
| | |
| Q1 | Q1 |
| Median | Median |
| Q3 | Q3 |
| Maximum | Maximum |

8. Create a box plot for the word counts you found for Shakespeare **AND** Harry Potter. Make sure to label your number line. (hint: make a stacked box plot)

9. Looking at the box plots and the mean, would you agree that there has been a "dumbing down" of America's youth over the past decades? Support your answer with numerical data you found in steps #4-8.

^{10.} Do you believe the comparison above could help you conclude the word counts for *ALL* Harry Potter and Shakespeare Literature? Why or why not?

Got Friends?

149

163

170

Is there a difference between the number of programmed numbers in the number of programmed telephone numbers in girls' cell phones and the number in boys' cell phones?

1. Do **you** think there is a difference? Why or why not?

When Mrs. Causey, an AP Statistics teacher at Olviedo High School in Seminole County, Florida, polled her students, she got the following data:

199

| Males | 5 | 20 | 26 | 40 | 46 | 47 | 49 | 50 | 51 |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 51 | 56 | 57 | 60 | 61 | 68 | 71 | 72 | 72 | 73 |
| 74 | 75 | 82 | 82 | 84 | 86 | 97 | 100 | 100 | 104 |
| 104 | 106 | 124 | 171 | 205 | 207 | 232 | 360 | | |
| | | | | | | | | | |
| Females | 20 | 46 | 50 | 58 | 62 | 65 | 70 | 72 | 72 |
| 80 | 86 | 87 | 88 | 90 | 92 | 94 | 94 | 109 | 112 |
| 114 | 116 | 122 | 125 | 129 | 137 | 137 | 138 | 142 | 142 |

204

249

2. Find the 5 number summary for both boys and girls:

186

| Males | Females |
|---------|---------|
| Minimum | Minimum |
| Q1 | Q1 |
| Median | Median |
| Q3 | Q3 |
| Maximum | Maximum |

3. Create side-by-side (stacked) box plots for the two sets of data above.

4. Compare the box plots. Do you notice a difference in their shape, center, and spread?

5. It is important that you have "data integrity". For example, it is important that data be reported accurately and truthfully. Do you think that this is the case here? Do you see any suspicious observations?

6. Can you think of any reason someone might make up a response or "stretch the truth" in reporting his or her number of programmed telephone numbers?

7. If you DO see a difference between the two groups, can you suggest a possible reason for that difference?

8. Do you think that a study of cell phone programmed numbers for a 7th grade math class would yield similar results? Why or why not?

Travel Times to Work

How long does it take you to get from home to school?

Here are the travel times from home to work in minutes for 15 workers in North Carolina, chosen at random by the Census Bureau:

| 5 | 10 | 10 | 10 | 10 | 12 | 15 | 20 | 20 | 25 | 30 | 30 | 40 | 40 | 60 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|

1. Find the mean of the North Carolina travel times.

Here are the travel times in minutes of 19 randomly chosen New York workers:

- 2. Find the mean of the New York travel times.
- 3. Compare the two means. Which state has a longer travel time when comparing the means?
- 4. Find the median of the North Carolina and New York travel times.

Median of North Carolina = _____ Median of New York= _____

- 5. Compare the medians. Which state has a longer travel time when comparing the medians?
- 6. Looking at the New York travel times, which number(s) affect the mean, but not the median?

7. Find the Mean Absolute Deviation (MAD) of the North Carolina and New York travel times. What does the MAD tell you about each set of data?

North Carolina: a) Find the distance that each value is away from the mean.

b) Total these values.

c) Divide by the total numbers of values in the set

<u>New York</u>: a) Find the distance that each value is away from the mean

b) Total these values.

c) Divide by the total numbers of values in the set

8. Overall, which measure of center best describes travel time to work, the mean or the median? Why?